Reply to "Comment on 'Restricted curvature model with suppression of extremal height'"

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We study the self-flattening surface of the restricted curvature model in one and two dimensions. The values of the roughness exponent are close to but slightly less than the prediction $\alpha = D \alpha_0 / (D + \alpha_0)$ of the preceding Comment where α_0 is the roughness exponent measured from the surface width *W* in the restricted curvature model and *D* is the dimension of the substrate. A possible explanation of the small discrepancy is discussed.

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The author of the preceding Comment [1] proposes a conjecture about the roughness exponent α ,

$$\alpha = \frac{D\alpha_0}{D + \alpha_0},\tag{1}$$

for the surface observed in our restricted curvature (RC) model with the global suppression (self-flattening RC model) [2], where α_0 is the roughness exponent measured from the surface width *W* in the ordinary RC model [3] without suppression and *D* is the dimension of the substrate.

In a previous study [2], we found $\alpha \approx 0.56$ from a direct measurement of saturated surface width up to the system size L=1024. We obtain one more data point of the saturated width for system size L=2048 and we calculate the effective exponent α_{eff} defined by

$$\alpha_{eff} = \frac{\ln[W^2(2L)/W^2(L)]}{2\ln(2)},$$
(2)

where W(L) is the surface width in the stationary state regime. Figure 1 shows the effective exponent α_{eff} as a function of 1/L for $L=32,64,\ldots,2048$. There is a little increment of α_{eff} as a function of system size L. The least square fit of $W^2 \sim L^{2\alpha}$ for all the data gives $\alpha \approx 0.57$ as shown in Fig. 1(a), but we can estimate that α is near to or larger than 0.58 as 1/L approaches zero. It is still slightly lower than 0.6 of Eq. (1). Since the effective size is proportional to L^{δ} , with $\delta \approx 0.42$ the effective size of our system might not be large enough to allow an accurate estimate of α . Thus, we cannot say that our numerical data exclude the possibility of α =0.6.

In the preceding Comment, the author assumes the stationary surface width $W(l) \sim W_0(l) \sim l^{\alpha_0}$ for a block of lateral size *l* and counts the entropic reduction due to the suppression where W_0 is the surface width of the ordinary RC model. He assumes that the short range height fluctuation behavior is described by the ordinary RC model with $\alpha' = \alpha_0$. It might be a reasonable guess in the Edwards-Wilkinson type self-flattening surfaces [4]. However, we do not have any numerical evidence about that in our anomalous roughening model [5,6] where the average step height (the average height difference between the nearest neighbors) at the saturated regime depends on the effective system size. Since the height difference correlation function G(r,t) for late times $(t \ge L^z)$ follows $G_s(r) \sim r^{2\alpha'} g_s(r/L^{\delta})$, the stationary surface width W(l) is proportional to $l^{\alpha'}$ for a block of lateral size $l \approx L^{\delta}$. Following the argument of the preceding Comment, one can obtain

$$\alpha = \frac{D\alpha'}{D+\alpha'}.$$
(3)

We get $\alpha' \approx 1.33$ [2] from the numerical data of the correlation function so that $\alpha = \alpha'/(1 + \alpha') \approx 0.57$ in D = 1. Our numerical results of the exponents are consistent with Eq. (3). More studies are required on the question that whether α' is equal to α_0 or not.

It is interesting to test the validity of Eq. (1) for higher dimensions. The ordinary unsuppressed RC model has $\alpha = \alpha' = 1$ and z = z' = 4 in D = 2 [3]. We also generalize the self-flattening RC model for D = 2 dimensions and measure both surface width and height difference correlation function. Our numerical estimates for the self-flattening RC model with the system sizes L = 24,32,48,64,96 are

$$\alpha \approx 0.65, \quad \alpha' \approx 0.96, \quad z \approx 2.8, \quad \text{and} \quad z' \approx 4.0$$
 (4)

in D=2. The value of α is slightly less than 2/3 of the Park's prediction. It seems that our data including the results of D



FIG. 1. (a) $W^2(L)$ for the systems of sizes $L = 16,32, \ldots, 2048$. The solid line is the least square fit of the form $W^2(L) \sim L^{2\alpha}$ with $2\alpha = 1.14$. (b) Effective roughness exponents $\alpha_{eff}(L)$ against 1/L.

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=2 are close to but slightly less than the values of Eq. (1). However, due to the finite size effect, we conclude that our data are roughly consistent with the prediction.

The correlation length increases with a power law $t^{1/z'}$ until it reaches the value proportional to L^{δ} at time $t_s \sim L^z$ where δ is the "window exponent" satisfying the relation δ

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 $=z/z' = \alpha/\alpha'$. Numerically we obtain that z' is very close to z_0 of the ordinary RC model. An unsolved question is whether the value of z' from the correlation function is the same as z_0 or not in our model.

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